

Total marks – 120 marks

Attempt Questions 1 – 10

All questions are of equal value

Answer the questions on your own paper or writing booklet, if provided. Start each question on a new page.

**STRATHFIELD GIRLS HIGH  
SCHOOL****2008****Higher School Certificate  
Trial Examination**

# Mathematics

**General Instructions**

- **Reading time – 5 minutes**
- Working time – 3 hours
- Board approved calculators may be used.
- Write using black or blue pen
- A table of standard integrals is provided at the back of the paper
- All necessary working should be shown in every question
- Write your student number and/or name at the top of every page

**Total marks – 120**

- Attempt Questions 1 – 10
- All questions are of equal value

**This paper MUST NOT be removed from the examination room**

STUDENT NUMBER/NAME: .....

**Total marks – 120 marks****Attempt Questions 1 – 10****All questions are of equal value**

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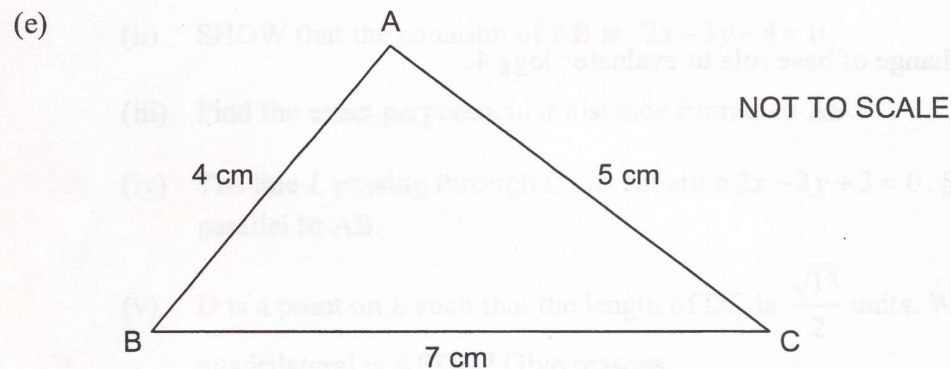
**Question 1 (12 marks)** Use a SEPARATE page or writing booklet**Marks**

(a) Evaluate:  $\sqrt[3]{e^{2.4}} - 1$  correct to 5 significant figures. 2

(b) Given:  $\frac{5}{\sqrt{3}-1} = a\sqrt{3} + b$ , find the values of  $a$  and  $b$ . 2

(c) Solve, giving your answer(s) in exact form:  $2x^2 - 5x - 4 = 0$ . 2

(d) Find a primitive of:  $\frac{1}{x^2} + \frac{1}{x}$ . 2



In the diagram above, find the size of the largest angle.  
Give your answer correct to the nearest degree.

2

(f) Simplify:  $\frac{5}{m-2} - \frac{2}{m-3}$ . 2



**Question 2** (12 marks) Use a SEPARATE page or writing booklet**Marks**(a) Differentiate with respect to  $x$ :

(i)  $\frac{\cos x}{x-1}$ .

2

(ii)  $(3x^2 - 7)^5$ .

2

(b) Solve:  $|2x - 3| < 1$ .

2

(c) (i) Find:  $\int \frac{x}{x^2 + 2} dx$ .

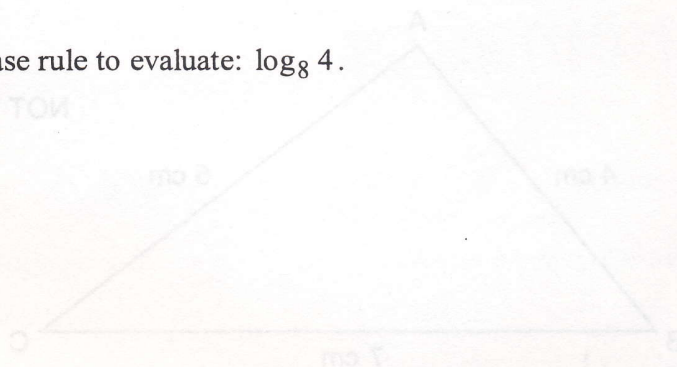
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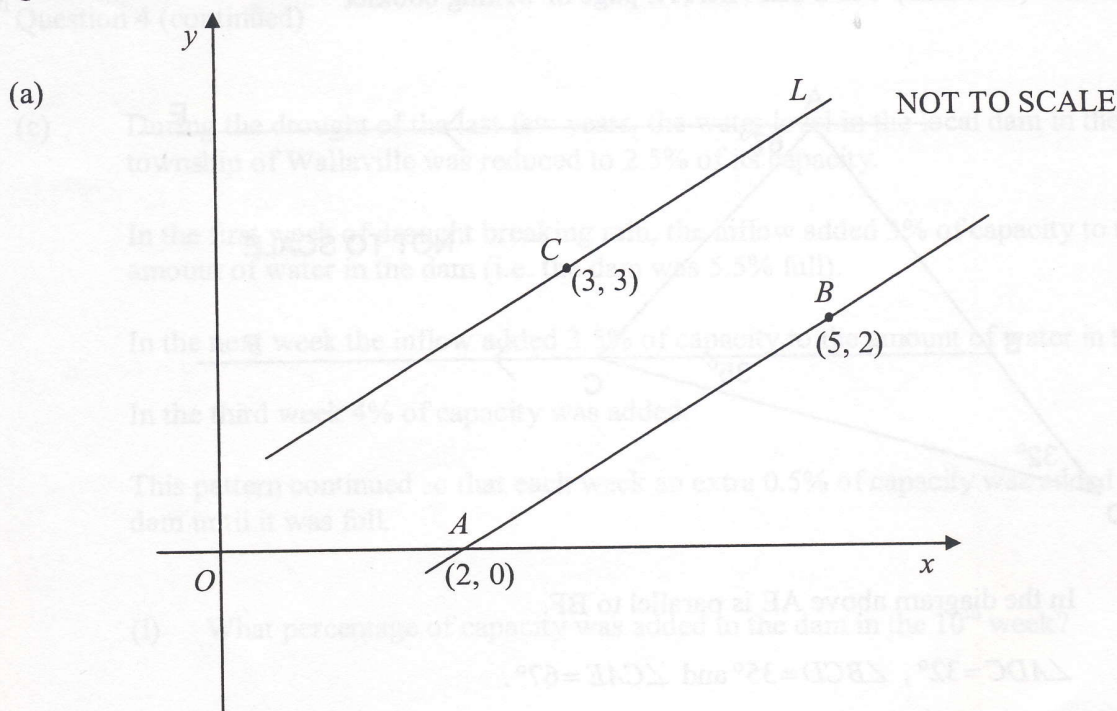
(ii) Evaluate:  $\int_0^{\frac{2\pi}{3}} \sin 2x dx$ .

3

(d) Use the change of base rule to evaluate:  $\log_8 4$ .

1

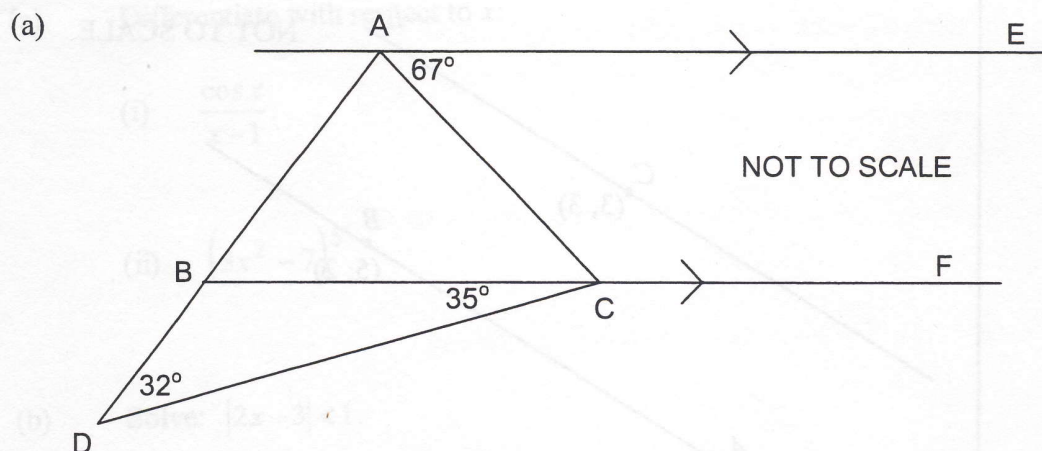


**Question 3** (12 marks) Use a SEPARATE page or writing booklet**Marks**

In the diagram above the points  $A(2,0)$ ,  $B(5,2)$  and  $C(3,3)$  are shown.  
Copy or trace the diagram onto your worksheet.

- |   |   |
|---|---|
| (i) Find the exact length of $AB$ .   | 1 |
| (ii) SHOW that the equation of $AB$ is $2x - 3y - 4 = 0$ .  | 1 |
| (iii) Find the exact perpendicular distance from $C$ to $AB$ .  | 1 |
| (iv) The line $L$ passing through $C$ has equation $2x - 3y + 3 = 0$ . Show that $L$ is parallel to $AB$ .                                  | 2 |
| (v) $D$ is a point on $L$ such that the length of $DC$ is $\frac{\sqrt{13}}{2}$ units. What type of quadrilateral is $ABCD$ ? Give reasons. | 1 |
| (vi) Calculate the area of $ABCD$ .   | 1 |
|   |   |
| (b) Solve: $2\cos A = -\sqrt{3}$ , for $0 \leq A \leq 2\pi$ .   | 2 |
|   |   |
| (c) Find the equation of the tangent to the curve $y = x^2 \ln x$ at the point $P$ on it where $x = e$ .                                    | 3 |



**Question 4** (12 marks) Use a SEPARATE page or writing booklet**Marks**

In the diagram above  $AE$  is parallel to  $BF$ .

$\angle ADC = 32^\circ$ ,  $\angle BCD = 35^\circ$  and  $\angle CAE = 67^\circ$ .

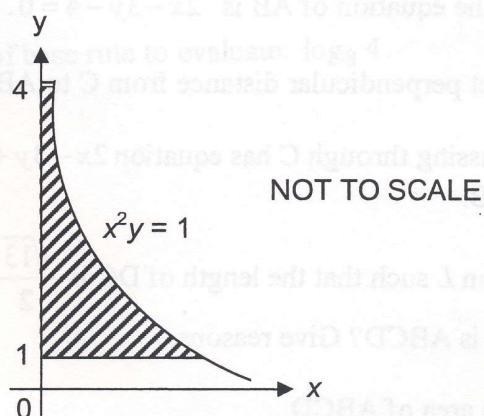
(i) Show that  $\triangle ABC$  is isosceles.

3

(ii) Find the size of  $\angle BAC$ .

1

(b)



The shaded region above shows the area bounded by the graph  $x^2 y = 1$ , ( $x > 0$ ), the  $y$ -axis and the lines  $y = 1$  and  $y = 4$ .

Find the volume of the solid of revolution formed when the shaded region is rotated about the  $y$ -axis. Give your answer in exact form.

3

**Question 4 continues on the next page**

## Question 4 (continued)

Marks

- (c) During the drought of the last few years, the water level in the local dam in the township of Wallaville was reduced to 2.5% of its capacity.

In the first week of drought breaking rain, the inflow added 3% of capacity to the amount of water in the dam (i.e. the dam was 5.5% full).

In the next week the inflow added 3.5% of capacity to the amount of water in the dam.

In the third week 4% of capacity was added.

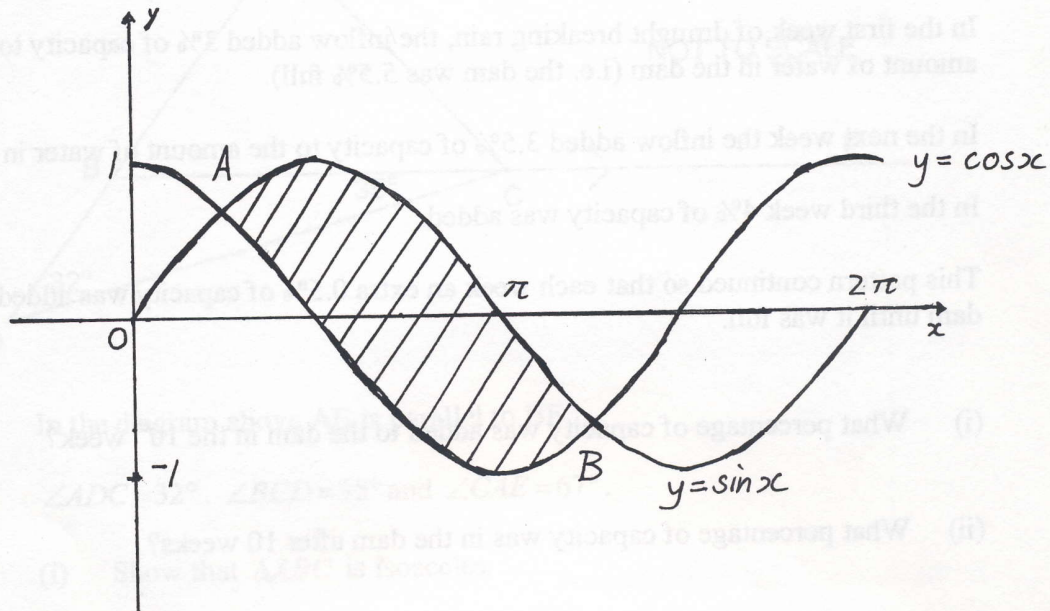
This pattern continued so that each week an extra 0.5% of capacity was added to the dam until it was full.

- |       |  |   |
|-------|--|---|
| (i)   | What percentage of capacity was added to the dam in the 10 <sup>th</sup> week? | 1 |
| (ii)  | What percentage of capacity was in the dam after 10 weeks?                     | 2 |
| (iii) | How many weeks would it have taken to fill the dam?                            | 2 |



**Question 5** (12 marks) Use a SEPARATE page or writing booklet**Marks**

(a)



The diagram shows the graphs  $y = \sin x$  and  $y = \cos x$ ,  $0 \leq x \leq 2\pi$ .  
The graphs intersect at A and B.

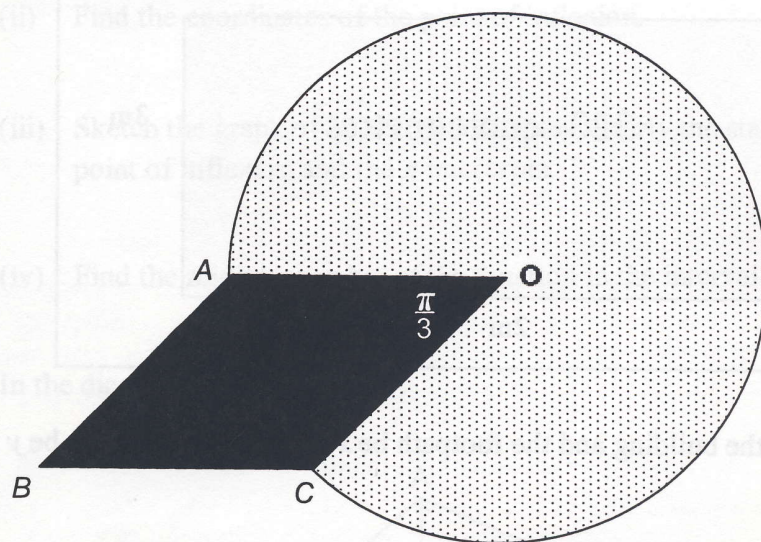
- (i) Show that point A has coordinates  $(\frac{\pi}{4}, \frac{1}{\sqrt{2}})$  and find the coordinates of B. 1
- (ii) Find the area enclosed by the two graphs. 3

**Question 5 continues on the next page**

## Question 5 (continued)

Marks

- (b) A concrete viewing platform is to be built at a mountain lookout. The platform is formed from a rhombus  $AOCB$  with side  $AO = 5\text{m}$  and  $\angle AOC = \frac{\pi}{3}$ , and the major sector of a circle centre  $O$ , radius  $AO$ . The concrete is 200mm thick. The platform is illustrated in the diagram below.

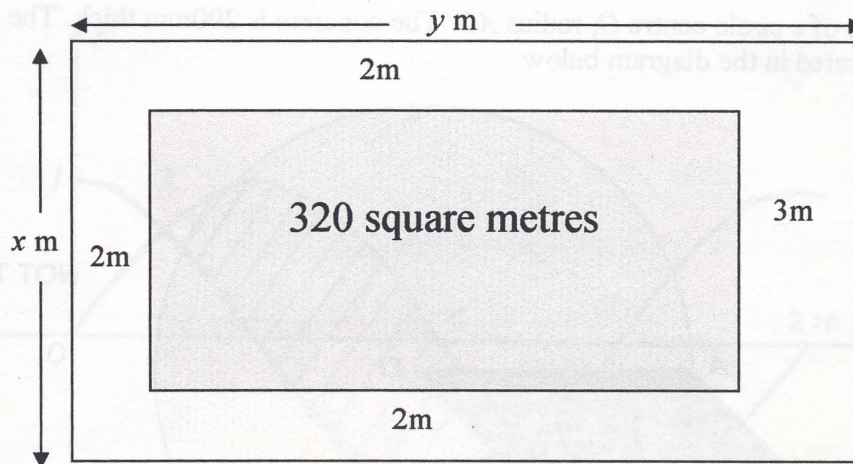


- (i) Show that reflex  $\angle AOC = \frac{5\pi}{3}$ . 1
- (ii) Calculate the area of the platform. 3
- (iii) Find the volume of concrete used to make the platform. 1
- (c) Given  $\tan A = \frac{\sqrt{15}}{7}$  and  $\pi \leq A \leq 2\pi$ , find the exact value of  $\operatorname{cosec} A$ . 3



**Question 6** (12 marks) Use a SEPARATE page or writing booklet**Marks**

- (a) An office block is being designed so that ground level is surrounded by a 2 metre width of concrete footpath at the sides and front and a 3 metre width at the front. The floorplan of the building must be 320 square metres as shown below.



Let the width of the building and the footpath be  $x$  metres and its length be  $y$  metres.

- (i) Show that the area  $A$  square metres of the floor and the footpath is:

$$A = x \left[ 5 + \frac{320}{(x-4)} \right]. \quad 2$$

- (ii) Find the dimensions of the smallest area  $A$  of the building and the footpath combined which fulfil these requirements. 5

(b) Evaluate:  $\sum_{x=0}^4 \left( \sin \frac{\pi x}{4} \right)$  2

(c) Given the infinite series:  $\frac{x}{3} + \frac{2x^2}{9} + \frac{4x^3}{27} + \dots$

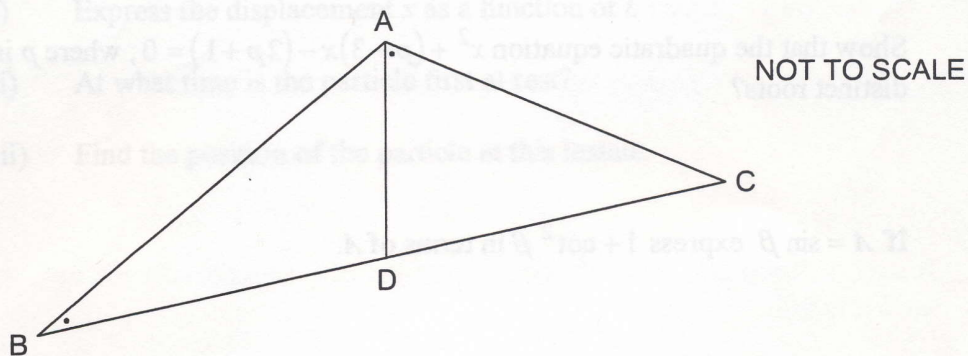
- (i) Show that it is a geometric series. 1
- (ii) Find the values of  $x$  such that the series has a limiting sum and find the sum (in terms of  $x$ ). 2

**Question 7** (12 marks) Use a SEPARATE page or writing booklet**Marks**(a) A function is defined as  $f(x) = x^3 - 3x^2$ .

(i) Find the coordinates of the stationary points and determine their nature.

**3**

(ii) Find the coordinates of the point of inflexion.

**1**(iii) Sketch the graph of  $y = f(x)$  indicating clearly the stationary points, the point of inflexion and the  $x$ -intercepts.**3**(iv) Find the minimum value of the function in the interval  $-2 \leq x \leq 3$ .**1**(b) In the diagram  $\angle CAD = \angle ABC$ .

Copy or trace the diagram onto your worksheet.

(i) Prove that  $\triangle CAD$  is similar to  $\triangle CBA$ .**3**(ii) Hence or otherwise show that  $AC^2 = CD \cdot CB$ .**1**



**Question 8** (12 marks) Use a SEPARATE page or writing booklet**Marks**

- (a) The number of worms,  $N$ , in a worm farm at time  $t$  weeks, is given by the formula:

$$N = N_0 e^{kt} \text{ where } N_0 \text{ and } k \text{ are constants.}$$

Initially there were 200 worms placed in the worm farm. After 2 weeks the number of worms had doubled.

- (i) Find the value of  $N_0$  and show that the value of  $k$  is 0.3466. 2
- (ii) How many worms were in the farm after 10 weeks? 1
- (iii) Find the rate of increase in the number of worms at 10 weeks. 2
- (iv) How many weeks would it take for the number of worms to increase by 900%? 2
- (b) Show that the quadratic equation  $x^2 + (p-3)x - (2p+1) = 0$ , where  $p$  is real, has real distinct roots? 3
- (c) If  $A = \sin \beta$  express  $1 + \cot^2 \beta$  in terms of  $A$ . 2



**Question 9** (12 marks) Use a SEPARATE page or writing booklet**Marks**

- (a) (i) Copy and complete the table below for the function  $f(x) = (x-1)^{-2}$ , giving the values correct to 3 significant figures. 1

$x$	2	2.5	3	3.5	4
$f(x)$					

- (ii) Using Simpson's Rule with 5 function values, find an approximate value for:

$$\int_2^4 (x-1)^{-2} dx.$$

2

- (b) A particle is moving in a straight line so that at time  $t$  seconds its displacement from the origin is  $x$  metres. Initially the particle is 1 metre to the left of the origin.

The velocity of the particle is given by  $v = \cos 2t - 1$ .

- (i) Express the displacement  $x$  as a function of  $t$ . 2
- (ii) At what time is the particle first at rest? 2
- (iii) Find the position of the particle at this instant. 1

- (c) The equation of a parabola is:  $2y = x^2 - 4x + 6$ .

- (i) Find the coordinates of the vertex, V, and the focus, S. 2
- (ii) Find the equation of the directrix. 1
- (iii) Draw a neat sketch of the graph of this parabola showing the information obtained in (i) and (ii) above. 1



**Question 10** (12 marks) Use a SEPARATE page or writing booklet**Marks**

- (a) Alana has borrowed \$17 000 to buy a new car. The interest on the loan is 18% per annum paid monthly. The loan is to be repaid in equal monthly instalments of \$ $P$  over a term of 5 years,

Let the amount owing on the loan after  $n$  months be  $\$A_n$ .

- |              |  |   |
|--------------|--|---|
| (i)          | Show that the amount $\$A_3$ owing after 3 months is given by:<br>$A_3 = \$\{(17000 \times 1.015^3) - P(1 + 1.015 + 1.015^2)\}$ .  | 2 |
| (ii)         | Write down a similar expression for the amount owing after 5 years.  | 1 |
| (iii)        | Calculate the monthly instalment \$ $P$ paid on the loan.  | 2 |
| (iv)         | Calculate the amount owing on the loan after 12 months.  | 1 |
| (v)          | After 12 months Alana lost her job and the bank agreed to change the terms of her loan. They agreed that she would make no repayments for 3 months, although interest would still be charged. At the end of the 3 months they would recalculate her instalments in order to repay the loan in full at the end of the same 5 years. |   |
| ( $\alpha$ ) | Calculate $A_{15}$ .   | 1 |
| ( $\beta$ )  | Calculate her new monthly instalment \$ $P$ .  | 1 |

- (b) A tank contains 200 litres of petrol. More petrol is pumped into the tank for 20 minutes, until it is full.

The volume flow rate  $R$  of petrol, in litres per minute is given by

$$R = 4(20 - t).$$

- |       |   |   |
|-------|---|---|
| (i)   | Find a formula for the volume $V$ of petrol in the tank after $t$ seconds where $t \leq 20$ . | 2 |
| (ii)  | How many litres of petrol were in the tank when it was full?                                  | 1 |
| (iii) | How long did it take to half fill the tank?   | 2 |

**End of paper**



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## STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left( x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left( x + \sqrt{x^2 + a^2} \right)$$

NOTE :  $\ln x = \log_e x, \quad x > 0$

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STUDENT NUMBER/NAME .....